N11 • Manipulating surds

Mathematical goals

To enable learners to:

- identify equivalent surds.

To develop learners' ability to:

- simplify expressions involving surds.

Starting points

Learners should understand what a square root is and be able to remove brackets correctly.

Materials required

For each learner you will need:

- Sheet 1 – *True or false?*;
- mini-whiteboard.

For each small group of learners you will need:

- Sheet 2 – *Show that* (cut into strips);
- a selection from Card set A – *Equivalent surds*;
- Card set B – *Surd dominoes* (to make larger dominoes, this sheet could be enlarged onto A3 card).

Time needed

At least 1 hour.

Suggested approach

Beginning the session

Give each learner a copy of Sheet 1 – *True or false?* and ask learners to use calculators to determine whether the statements in the first half of the sheet are true or false and then fill in the = or ≠ in the expressions at the bottom.

Whole group discussion (1)

Discuss the meaning of the $\sqrt{\ }$ sign and why we often leave it in expressions rather than using decimal approximation. Reinforce the generalisations at the bottom of Sheet 1 by linking with the numerical examples on the page.

Use mini-whiteboards to check some simple manipulations such as:

$$\left(\sqrt{2}\right)^4 \qquad 3\sqrt{7} \times 2\sqrt{7} \qquad \sqrt{3}\left(\sqrt{2} - 6\right)$$

Working in groups (1)

Give each pair of learners one of the strips from Sheet 2 – *Show that.* Ask them to provide the working. As each pair finishes a strip, check their working and give them another. When all pairs have done at least three strips, give a selection of cards from Card set A *Equivalent surds* to each pair and ask them to discuss why each surd can be simplified in that way.

Whole group discussion (2)

Ask each pair of learners to explain one of their simplifications. Explain that 'expressing a surd in its simplest form' means that the number inside the surd should be as small as possible. Discuss strategies e.g. finding the highest factor that is a square number. Practise a few examples on mini-whiteboards.

Working in groups (2)

Give each pair of learners Card set B – *Surd dominoes* and ask them to make a domino chain. Note on the board any problems or difficulties that learners encounter.

For learners who find the manipulation of surds easy, some of the dominoes could be removed and replaced with blanks for them to complete. If any learners finish early they could be encouraged to add some dominoes of their own.

Reviewing and extending learning

Discuss together all the points that have been noted on the board. Then put the addition table and multiplication table below on the board for completion by learners working in pairs or on their own.

+		
	$3\sqrt{3} + 2\sqrt{2}$	$2\sqrt{3} + 3\sqrt{2}$
	$2\sqrt{3}$	$\sqrt{3} + \sqrt{2}$

×		
	$6\sqrt{6}$	18
	12	$6\sqrt{6}$

Learners could then write their own tables and pass them to other learners for completion.

What learners might do next

Use equivalent surds to simplify exact form solutions from quadratic equations.

Rationalise the denominator of surds.

Further ideas

The ideas in this session can be adapted for learning and practising a variety of skills, e.g. fractions, decimals, indices, logarithms and factorising.

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N11 Sheet 1 – *True or false?*

$\sqrt{9} + \sqrt{4} = \sqrt{13}$	$\sqrt{2} = 1.4142136$
$\sqrt{3} \times \sqrt{3} = 3$	$\sqrt{2} = 1.414213562$
$\sqrt{9} \times \sqrt{4} = \sqrt{36}$	$\sqrt{2} = 1.4142$
$\sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2}$	$\sqrt{9} - \sqrt{4} = \sqrt{5}$
$4\sqrt{3} + 5\sqrt{3} = 9\sqrt{3}$	$\frac{\sqrt{36}}{\sqrt{4}} = \sqrt{9}$
$2\sqrt{3} \times 4\sqrt{3} = 8\sqrt{3}$	

a, b, x and y are positive integers.

Fill in $=$ or $\neq$

$\sqrt{a} + \sqrt{b}$	$\sqrt{a + b}$	$\sqrt{x^2} + \sqrt{y^2}$	$x + y$
$\sqrt{a} - \sqrt{b}$	$\sqrt{a - b}$	$\sqrt{x^2 + y^2}$	$x + y$
$\sqrt{a} \times \sqrt{b}$	$\sqrt{ab}$	$\sqrt{x^2} \times \sqrt{y^2}$	xy
$\sqrt{\frac{a}{b}}$	$\frac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\frac{x^2}{y^2}}$	$\frac{x}{y}$

N11 Sheet 2 – *Show that*

Show that $\left(\sqrt{3}+2\right)\left(\sqrt{3}+4\right)=11+6\sqrt{3}$

1

Show that $\left(\sqrt{5}-2\right)\left(\sqrt{5}+3\right)=\sqrt{5}-1$

2

Show that $\left(2\sqrt{3}-1\right)\left(2\sqrt{3}+1\right)=11$

3

Show that $\sqrt{18}\times\sqrt{2}-\sqrt{2}\times\sqrt{50}=-4$

4

Show that $\sqrt{3}+2\left(3\sqrt{3}-1\right)=7\sqrt{3}-2$

5

Show that $\left(\sqrt{3}\right)^2+\left(3\sqrt{2}\right)^2=21$

6

N11 Card set A – *Equivalent surds*

$\sqrt{18} = 3\sqrt{2}$	$\sqrt{72} = 6\sqrt{2}$
$\sqrt{45} = 3\sqrt{5}$	$\sqrt{50} = 5\sqrt{2}$
$\sqrt{32} = 4\sqrt{2}$	$\sqrt{90} = 3\sqrt{10}$
$\sqrt{28} = 2\sqrt{7}$	$\sqrt{54} = 3\sqrt{6}$
$\sqrt{20} = 2\sqrt{5}$	$\sqrt{8} = 2\sqrt{2}$
$\sqrt{128} = 8\sqrt{2}$	$\sqrt{75} = 5\sqrt{3}$
$\sqrt{63} = 3\sqrt{7}$	$\sqrt{48} = 4\sqrt{3}$

N11 Card set B – *Surd dominoes*

Note: there are twenty dominoes in the set

$\sqrt{8}$	$\sqrt{18}+3\sqrt{2}$	$3\sqrt{2}$	$\sqrt{90}$
$\sqrt{80}$	$\frac{\sqrt{50}}{5}$	$6\sqrt{2}$	$2\sqrt{3}\times 5\sqrt{3}$
$\sqrt{2}$	$\frac{\sqrt{72}}{\sqrt{3}}$	$12\sqrt{6}$	$\sqrt{40}\times\sqrt{90}$
$3\sqrt{10}$	$\frac{\sqrt{54}}{\sqrt{6}}$	60	Finish
40	$\frac{\sqrt{84}}{2}$	$2\sqrt{6}$	$\sqrt{8}+\sqrt{2}$
9	$\frac{8+\sqrt{48}}{4}$	$5+2\sqrt{6}$	$\left(\sqrt{3}\right)^4$
30	$\sqrt{8}\times\sqrt{50}$	3	$\sqrt{128}$
Start	$\sqrt{3}\left(2\sqrt{3}-1\right)$	20	$\sqrt{10}\times\sqrt{8}$
$2+\sqrt{3}$	$3\sqrt{2}\times 4\sqrt{3}$	$8\sqrt{2}$	$2\sqrt{5}\times 4\sqrt{5}$
$6-\sqrt{3}$	$2\sqrt{2}$	$\sqrt{21}$	$\left(\sqrt{3}+\sqrt{2}\right)^2$